TONBRIDGE SCHOOL

Scholarship Examination 2009

MATHEMATICS I

Tuesday 28th April 2009
11.15 a.m.

Time allowed: 1 hour 30 minutes

Answer as many questions as you can.
Questions 1 to 5 are worth 8 marks each;
Questions 6 to 9 are worth 15 marks each.

All answers must be supported by adequate explanation.
Calculators may be used in any question.

1. Solve the simultaneous equations:

$$4x+3y=65$$
$$3(x+5)+4(y-5)=63.$$

[8]

2. Consider the following sequence of sums.

Sum 1: $\dfrac{2}{1}-\dfrac{1}{2}=\dfrac{3}{2}=\dfrac{3}{1\times2}$

Sum 2: $\dfrac{3}{2}-\dfrac{2}{3}=\dfrac{5}{6}=\dfrac{5}{2\times3}$

$\overline{\quad\quad\quad\quad\quad}$ $n\times(n+1)$

$3,2,5,3$

$\dfrac{100}{3}=33.?$

$33\times3=99$

(a) Write out Sum 3 and Sum 4 in a similar form.

(b) Use the pattern in your answers to write down Sum 100.

(c) (i) For which n does Sum n have a numerator of 1805 in its answer?
   (ii) For which n does Sum n have a denominator of 1806 in its answer?

[8]

3. A 10 pence coin has a mass of 6.5 grams; 75% of its mass is copper and 25% is nickel. Copper costs £3.70 per kilogram; Nickel costs £17.50 per kilogram.

(a) What is the value of the metal in a 10 pence coin?

(b) If a 10 pence coin (of mass 6.5 grams) were made entirely of Nickel, what would be the value of the metal in the 10 pence coin?

(c) For what percentages of Copper and Nickel would the value of the metal in a 10 pence coin (of mass 6.5 grams) be exactly 10 pence?

[8]

4. The equation of a curved graph is given by $y=\dfrac{a^3}{x^2+4}$ where a is a constant.

(a) If $a=2$ and $x=5$, find the value of y.

(b) If $x=1.5$ and $y=6$, find the value of a.

(c) If $a=6$ and $y=12$, find the value of x.

$4,5,8,2$

[8]
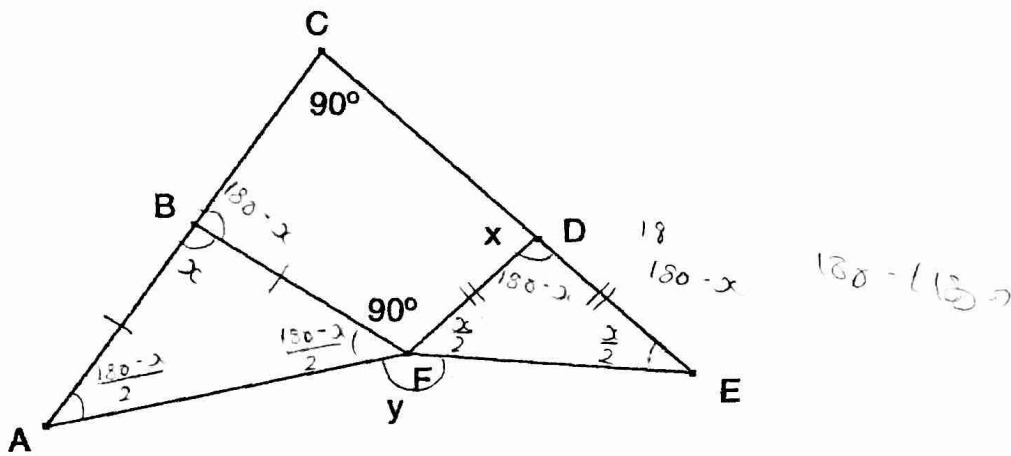
$\dfrac{111}{4}=27.75$ $\quad 111$

$=108$

27

5. In the figure (which is not drawn to scale!), triangles ABF and FDE are isosceles with AB = BF and FD = DE. ABC and CDE are straight lines and angles BCD and BFD are right angles as shown; angle CDF is x and angle AFE is y.

(a) If $x = 50°$, find y.

(b) Show that, whatever the value of x, the value of y is always the same.

[8]

6. A sector of a circle of radius 3 cm comprises a fraction x of the circle. (Thus a semicircle corresponds to $x = 0.5$ and a quadrant to $x = 0.25$.) If the edges of the sector are brought together to form a cone, its volume y is given by $y = 9\pi x^2 \sqrt{1-x^2}$.

(a) When $x = 0.5$, show that $y = 6.12$ (correct to 2 decimal places).

(b) By considering the sectors involved, explain why $y = 0$ when $x = 0$ and when $x = 1$.

(c) Find the values of y for $x = 0.2, 0.3, 0.4, 0.6, 0.7, 0.8, 0.85, 0.9, 0.95$.

(d) Choosing sensible scales, use your values to plot a graph of y against x.

(e) Use your graph to find two values of x that give a volume of 8 cm³.

(f) Which value of x gives the maximum volume?

[15]

7. *In this question, you should use calculations involving Pythagoras's Theorem and not measurements off a scale drawing.*

Points A, B, C, D have coordinates A(3,4), B(14,6), C(8,14), D(18,-16).

(a) By using Pythagoras in the triangle ABE where E is the point E(14,4), show that distance AB is $\sqrt{125}$.

(b) Give calculations to show that triangle ABC is isosceles.

(c) Find the area of triangle ABC.

(d) Give calculations to show that ABD is a right-angled triangle.

[15]

8. In the figure below, Region I consists of three-quarters of a circle, radius 6 cm, centre O, together with the right-angled triangle AOB. Region II is a rectangle, with two sides of length equal to AB and the other two sides of length x cm.
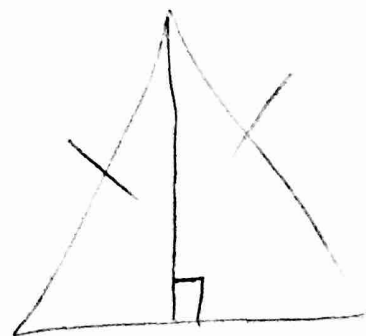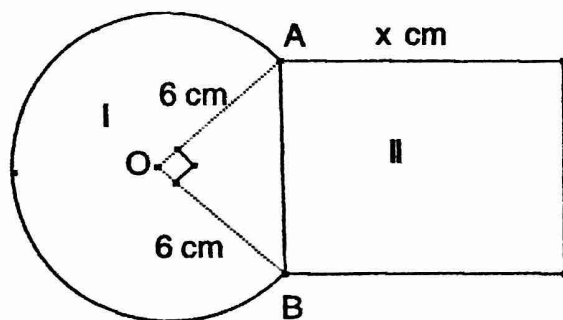
(a) Find x if regions I and II have the same area.

(b) Find x if regions I and II have the same perimeter.

[15]

9. This question is about the sequence of *triangular numbers* that starts
$T_1 = 1$, $T_2 = 3$, $T_3 = 6$, $T_4 = 10$, ...

(a) Spot the pattern and then write down the values of T_5, T_6, T_7, T_8, T_9 .

(b) (i) $T_1^2 + T_2^2$ is a triangular number: which one?
 (ii) $T_2^2 + T_3^2$ is a triangular number: which one?

(c) Use your answer to (b) to predict which triangular number is equal to $T_3^2 + T_4^2$.

(d) (i) Find x if $T_{10}^2 + T_{11}^2 = T_x$.
 (ii) Find y if $T_y^2 + T_{y+1}^2 = T_{81}$.

[15]